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ABSTRACT

The guide (one-quarter trigonometry course; two-quarter analytic geometry course) provides both subject matter and career preparation assistance for advanced mathematics teachers. It is arranged in vertical columns relating curriculum concepts in trigonometry and analytic geometry to curriculum performance objectives, career concepts and teaching activities, suggested teaching methods, and audio-visual and resource materials. Space is provided for teachers' notes which will be useful when the guide is revised. (EA)

TRIGONOMETRY

Career - Curriculum Guide

CAREER EDUCATION CENTER

HARLANDALE INDEPENDENT SCHOOL DISTRICT

102 GENEVIEVE

ANALYTICAL GEOMETRY

HARLANDALE INDEPENDENT SCHOOL DISTRICT

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TRIGONOMETRY
(a one quarter course)

CURRICULUM GUIDE

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The audio-visual materials listed in this guide have been assigned catalogue numbers by the Harlandale Independent School District audio-visual department, San Antonio, Texas.

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Mr. William H. Bentley - Director of Vocational Education

Miss Mary E. Daunoy - Secondary Consultant

Mr. Hamilton C. Dupont (deceased) - Head of Math Department

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DNS:ARE

Preface

Meaningful existence is the goal of life in today's world. Living takes on meaning when it produces a sense of self-satisfaction. The primary task of education must be to provide each individual with skills necessary to reach his goal.

When children enter school, they bring with them natural inquisitiveness concerning the world around them. Normal curiosity can be the nucleus which links reality to formal training if it is properly developed. A sense of continuity must be established which places education in the correct perspective. Communities must become classrooms and teachers resource persons. Skills such as listening, problem solving, following directions, independent thinking and rational judgement then can merge into daily living procedures.

In classrooms especially designed to form a bridge between school and the world of work, experiences must be developed. On campus performance in job tasks and skills, following a planned sequence of onsite visitation, will fuse information into reality. Practical relationships developed with those outside the formal school setting will provide an invaluable carry-over of learned skills.

Search for a rewarding life vocation is never easy. Without preparation it becomes a game of chance. With a deliberate, sequential, and planned program of development, decisions can be made based upon informed and educated judgements.

A full range career education program, K-12, will offer opportunities for participants to enter employment immediately upon completion of training, post secondary vocational-technical education, and/or a four-year college career preparatory program.

C. N. Boggess

C. N. Boggess, Superintendent
Harlandale Independent School District

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PHILOSOPHY

The educational needs of any community are somewhat unique. This was certain to have been one of the guiding principles used when our forefathers set up local control for public schools. Accordingly, the philosophy of the Harlandale school system is to serve the educational needs of all its citizens as evidenced by adult classes, government sponsored retraining programs, vocational courses, etc. The mathematics department follows this philosophy in planning a program best suited to the needs of our students.

The past decade has proven the need for the new emphasis on the importance of the study of mathematics. Usefulness of mathematics in many fields of learning and endeavor, long thought to be free of mathematics, is now an accepted reality. Also, basic principles must be understood in order that mathematical systems can be devised to describe new human or mechanical activities, as they come to pass.

As mathematics continues to grow this must, of necessity, result in the addition of new symbols, terms, topics, and new approaches. The changing times will make some older topics and methods obsolete. To meet the mathematical needs of our students and to assist teachers in their instruction, the mathematics department has prepared this mathematics guide. Any given faculty consists of personnel with different degrees of training, experience, local tenor, and understanding of student needs. Hence, the desirability for some guiding criteria. In addition, we feel all students should consider career planning as a major facet of their education. Thus it follows that they will need some exposure to the different mathematical requirements of the varied career fields. In part, it is the purpose of this guide to assist the teacher in providing appropriate instruction to meet such needs.

The department feels that the most important basic guide for any mathematics course is the textbook, and careful care is taken in the selection of this book. Not only is the textbook an important guide for the teacher but it is also desirable for the student to learn the use of a textbook as a guide and important tool for learning.

Therefore, the plan of this mathematics guide is not to rewrite the textbook but to improve on it. Generally, the plan is to implement, where desirable, the textbook coverage, describe supplementary material that is needed and make suggestions on methods, procedures, order of coverage, etc.

Mathematics is a thoughtful, creative and intellectually stimulating subject. The enthusiasm and interest of the teacher in the subject is the best atmosphere for creating student enthusiasm for mathematics. This guide is planned to help foster this enthusiasm and in no way infringes on the academic freedom of the teacher.

It is hoped that the guide will prove helpful in understanding the basic standards, improving instruction, and developing the desired uniformity for the classes in each area of study. Finally, the guide should serve as the nucleus for a continuing effort to improve mathematics instruction.

Mr. Hamilton C. Dupont (deceased) - Head of Math Department
Harlandale Independent School District

TRIGONOMETRY

Overview

Trigonometry, which extends concepts of algebra and geometry, is designed for students in a college-preparatory program. Trigonometry prepares students for Analytic Geometry, Elementary Analysis, Physics, Calculus with Analytic Geometry, engineering, and later college courses. This course emphasizes the role of functions--both circular and trigonometric--in developing trigonometric concepts: assists the pupil in developing understandings of the ideas associated with angles, triangles, and vectors; and provides applications of trigonometric concepts. Basic content of the course includes areas of study presented in the sequence suggested.

Goals

To acquire an understanding of the development of the circular functions and their properties

To establish the trigonometric function as a special type of circular function

To develop an appreciation for the study of functions

To develop an ability to handle situations involving applications of trigonometric functions

To improve the student's appreciation of the structure and unity of mathematics

To acquire a higher level of sophistication in mathematics so that the student is prepared for study of analysis or calculus

To improve the student's understanding of the relationship between various careers and trigonometric concepts

TRIGONOMETRY

The following outline is built upon a "conceptual ladder" (concepts from 'easiest to hardest') for trigonometry. The outline corresponds to the outline found in the curriculum concepts of this guide. The page numbers refer to the present textbook being used in trigonometry by the Harlandale Independent School District. (Vance, Elbridge P., *Trigonometry*. Reading, Massachusetts: Addison-Wesley, 1971.)

I. Functions and Relations
   - A. Domain - p. 24
   - B. Range - p. 24

II. Circular Functions
   - A. Unit Circle - pp. 19-20
   - B. Cosine - p. 35
   - C. Sine - p. 35
   - D. Tangent - p. 36
   - E. Meaning of θ - p. 34
   - F. Signs in Quadrants - p. 36
   - G. Reciprocal Functions - p. 39
   - H. Graph of Sine and Cosine - pp. 42-43
   - I. Values at Selected Points - p. 42
   - J. Identities - pp. 53-56

III. Functions Involving More Than One Argument
   - A. Cos $(\alpha-\beta)$ - p. 61
   - B. Special Reduction Formulas - pp. 63-65
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   - D. General Reduction Formulas - pp. 69-71
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   - G. Circular Functions - pp. 81-84

IV. Complex Numbers
   - A. Definition - pp. 102-103
   - B. Modulus - p. 104
   - C. Conjugates - p. 103
   - D. Operations - pp. 104-107

V. Applications of Circular Functions to Angles
   - A. Angles and Arcs - pp. 144-149
   - B. Circular Functions of Angles - pp. 149-153
   - C. Geometric Use of Angles - pp. 153-161
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   - F. Oblique Triangles
     1. Law of Sines - pp. 164-166
     2. Law of Cosines - pp. 173-174
     3. Applications - pp. 175-181

VI. Inverse Circular Functions

  A. Arc Sin θ, Arc Cos θ, and Arc Tan θ - pp. 111-115

  B. Operations - pp. 115-119

VII. Graphs of Functions

  A. Periodic Functions

    1. Amplitude - p. 126

    2. Period - p. 126

    3. Phase Shift - p. 128

  B. $y = a \sin kx$ - pp. 126-128

  C. $y = a \sin (kx + b)$ pp. 128-130

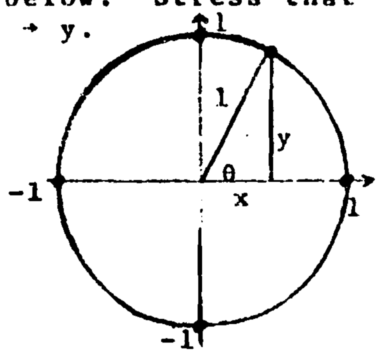
CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
I. Functions and Relations A. Domain	I. THE STUDENT SHOULD BE ABLE TO: A. Given ten functions, determine with 80% accuracy the domain of each function.	Concept Relationship of the domain and range of a quadratic function to the number of new cars sold as predicted by an economist. Performance Objective Given the necessary quadratic equation, calculate the highest number of new cars which would be sold according to the number of extras on each car.
B. Range	B. Given ten functions, determine with 80% accuracy the range of each function.	General Information Students wishing extra activities should be encouraged to research other careers in the economic field. They may wish to compile and analyze material on work done by the economist. Economics is the largest of the basic social science fields. Approximately 33,000 economists were employed in 1970 with salaries ranging from $6,548 as a start to over $23,000 yearly. A bachelor's degree with a major in economics is sufficient for many beginning jobs. Graduate work is very important for advancement. Teaching Activity Economists employed by car manufacturers have determined that the number of new cars

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
I. A. Define a function as a set of ordered pairs (x,y) none of which have the same first element. Define a relation from X to Y as a set of ordered pairs (x,y) such that to each $x \in X$ there corresponds at least one $y \in Y$. Stress that any function is a relation, but there exist many relations which are not functions. Stress that the domain is the x coordinate in the ordered pair (x,y). Give examples. B. Stress that the range of the function is the y coordinate of the ordered pair (x,y) after the rule of the function has been performed. Work examples given the domain. Allow the students to find the range of sample problems.	Harlandale Audio-Visual Center Curriculum Introduction to Functions; filmstrip -- Z - 41 Career Economist; magnetic tape -- MT - 310 (side 2) For Additional Information on Economic Careers American Economic Association 1313 21st Avenue South Nashville, Tenn. 37212 The Foreign Service in the Seventies, U.S. Department of State, Publication 8535 Washington, D.C. 20520. Free. The International Developer (Economist), Professional Talent Search, Office of Personnel and Manpower, Agency for International Development, Washington, D.C. 20523. Free.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		sold highly correlates to the number of extras on the car. Experience is usually the main requirement for determining which extras will sell a car. An economist knows that if x is the number of extras he advises be put on a car, then from past experience he knows he can expect to sell $f(x)= 40 + 8x - x^2$ cars during the model year. The car which he wishes to sell comes with anywhere from 0 to 8 extras (such as power steering, radi air-conditioning, and so on). Therefore, the domin of this function is the set $x \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$. $f(0)=40$ $f(5)=55$ $f(1)=47$ $f(6)=52$ $f(2)=52$ $f(7)=47$ $f(3)=55$ $f(8)=40$ $f(4)=56$ The range of this function is the set $f(x) \in \{40, 47, 52, 55, 56\}$. The above calculations indicate to the economist that he should advise the car manufacturer that he can expect to sell the highest number of cars which contain 4 extras.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
II. Circular Functions A. Unit Circle	II. THE STUDENT SHOULD BE ABLE TO: A. Define a unit circle as a circle with its center at the origin of a coordinate plane and a radius of 1.	Concept Relationship of the study of circular functions to the task of locating the position of a town on a map by a cartographer. Performance Objective Given the longitude and latitude of two different towns and the distance of each town from a town not on a map, determine the latitude and longitude of the town not on the map.
B. Cosine & C. Sine	B. Define cos θ as x where x is the abscissa of $P(\theta)$ on the unit circle. C. Define sin θ as y where y is the ordinate of $P(\theta)$ on the unit circle.	General Information Students wishing extra activities should be encouraged to research other careers related to geography. Examples: 1. Cartographer 2. Map cataloguer 3. Title geographer.
D. Tangent	D. Define tan θ as y/x where x is the abscissa of $P(\theta)$ and y is the ordinate of $P(\theta)$ on the unit circle.	Employment for geographers is expected to be favorable through the 1970's. Nationally the salaries for geographers range from $6,548 to over $20,000.
E. Meaning of θ	E. Define θ in regard to the study of trigonometry as an angle formed on the unit circle.	Geographers must receive at least a bachelor's degree with expection of obtaining a higher degree. Teaching Activity Suppose you want to place the town of Morton on a map. Its

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
II. A. Define a unit circle as one with its center at the origin and a radius of 1. Draw on the board several circles which have different lengths for their radii. Point out that all these circles could be considered unit circles. B.&C. Draw a unit circle on the board. Label the drawing as shown below. Stress that cos: $\theta \to x$ and sin: $\theta \to y$.  D. Again using a unit circle define the tangent of θ as y/x. Make sure the students note the relationship between the three functions. $\tan = \{(\theta, z) \mid z = \tan\theta = \frac{\sin\theta}{\cos\theta}\}$ E. It is suggested that the students memorize the letters of the Greek alphabet because of their wide use in mathematics. The letter theta, θ, should be defined to denote arc length and the measure of an angle.	<u>For Additional Information On Geographical Careers</u> American Congress on Surveying and Mapping, 733 Fifteenth Street NW, Washington, D.C. 20005 American Society of Photogammetry, 105 North Virginia Avenue, Falls Church, Virginia 22046	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
F. F. Signs in Quadrants	F. Complete a chart with 80% accuracy which shows the signs of the sine, cosine, and tangent functions in each of the four quadrants.	latitude and logitude are not known, but surveyors report that it is 128 miles from the town of Smithsville, whos position is 42^0 North Latitude and 37^0 West Longitude. We know that Morton lies at a bearing of 36^0 northeast of Smithsville. At this latitude, a degree of longitude equals 50 miles. A degree of latitude has the same value it has at any point on the earth.
G. Reciprocal Functions	G. Define the csc $\theta = 1/y$, sec $\theta = 1/x$, and cot $\theta = x/y$ where x is the abscissa of $p(\theta)$ on the unit circle.	
H. Graph of Sine and Cosine	H. Draw with 80% accuracy the graph of the sine and cosine functions from 0 to 2π.	<u>Solution</u>:
I. Values at Selected Points	I. Complete a chart with 80% accuracy which exhibits the value of the sine and cosine at 0, $\pi/2$, π, $3\pi/2$, and 2π.	x= distance in mi. Morton lie north of Smithsville. y= distance in mi. Morton lie west of Smithsville. $x = 128 \sin 36^0$ $x = 75.2$ mi. $x = 1^0 4'$
J. Identities	J. Prove with 80% accuracy ten identities concerning circular functions.	$y = 128 \cos 36^0$ $y = 103.5$ mi. $y = 2^0 4'$ $43^0 4'$ North $34^0 6'$ West

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
F. After a board demonstration of $\sin \theta$, $\cos \theta$, and $\tan \theta$ in each of the four quadrants, request the students to make a chart of the signs of each function in each quadrant. G. Review the meaning of reciprocal functions in terms of the reciprocal of any number a as the number $1/a$ such that $a \cdot 1/a = 1$. Define the cosecant of θ as $1/\sin \theta$, the secant of θ as $1/\cos \theta$, and the cotangent of θ as $\cos \theta / \sin \theta$. H. Demonstrate the graph of the sine and cosine of θ from 0 to 2π. On a written exercise request the students to graph the sine and cosine of θ from -2π to 4π. I. Use the unit circle to present the sine, cosine, tangent, cotangent, secant, and cosecant of $\pi/2$, $\pi/3$, $\pi/4$, $\pi/5$, and $\pi/6$ in each quadrant. Suggest to the student to make a chart of these values to keep for a ready reference. J. Define and discuss the fundamental circular identities. Discuss the proofs of selected identities. Request and select several students to give proofs of identities at the board.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
III. Functions Involving More Than One Argument A. cos $(\alpha-\beta)$ B. Special Reduction Formulas C. Addition Formulas	III. THE STUDENT SHOULD BE ABLE TO: A. Given the values of α and β in ten problems, determine with 80% accuracy the value of $\cos(\alpha-\beta)$ by using the formula $\cos(\alpha-\beta) \equiv \cos\alpha \cos+\beta \sin\alpha \sin\beta$. B. Given ten problems concerning the use of special reduction formulas, determine with 80% accuracy the required reduction by use of the correct formula. C. Given ten problems which use the addition formulas in their solution, determine with 80% accuracy the required addition by use of the correct formula.	<u>Concept</u> Relationship of the study of trigonometric formulas to the job done by a lighting engineer. <u>Performance Objective</u> Given the necessary formula, calculate the amount of light needed to illuminate a billboard. <u>General Information</u> Students wishing extra activities should be encouraged to research other careers in lighting. Over 300,000 engineers were employed in now-manufacturing industries in 1970. Employment opportunities for engineers are expected to be favorable through the 1970's. New engineering graduates having the bachelor's degrees and no experience earned an average of \$10,400 a year. With more advanced degrees the starting salary jumped to \$16,000. Salaries for experienced engineers can be much higher. <u>Teaching Activity</u> Trigonometry may help the lighting engineer in determining the most effective lighting for a billboard advertisement. The

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
III. A. Demonstrate the proof of the formula $\cos(\alpha-\beta)\equiv\cos\alpha\cos\beta+\sin\alpha\sin\beta$ at the board. Select several students to show the use of this formula at the board. B. Derive each of the special reduction formulas at the board. Request that the students memorize each of the formulas. 1. $\cos(\pi/2-\beta)\equiv\sin\beta$ 2. $\sin(\pi/2-\beta)\equiv\cos\beta$ 3. $\tan(\pi/2-\beta)\equiv\cot\beta$ 4. $\cot(\pi/2-\beta)\equiv\tan\beta$ 5. $\cos(-\beta)\equiv\cos\beta$ 6. $\sin(-\beta)\equiv-\sin\beta$ 7. $\tan(-\beta)\equiv-\tan\beta$ C. Demonstrate the derivation of the general addition formulas at the board. Request that the students memorize each of the addition formulas. 1. $\cos(\alpha+\beta)\equiv\cos\alpha\cos\beta-\sin\alpha\sin\beta$ 2. $\sin(\alpha+\beta)\equiv\sin\alpha\cos\beta+\cos\alpha\sin\beta$ 3. $\sin(\alpha-\beta)\equiv\sin\alpha\cos\beta-\cos\alpha\sin\beta$ 4. $\tan(\alpha+\beta)\equiv\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ 5. $\tan(\alpha-\beta)\equiv\dfrac{\tan\alpha+\tan\beta}{1+\tan\alpha\tan\beta}$	Harlandale Audio-Visual Center <u>Career</u> <u>Careers in Television</u>; record with filmstrip -- PR - 1026	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
D. General Reduction Formulas	D. Given ten problems which use the general reduction formulas in their solution, determine with 80% accuracy the required reduction by use of the correct formula.	illumination on a surface at distance r from the source is given by the equation $E = \frac{I \cos \theta}{r^2}$ where I is the intensity of the source, and θ is the angle between direction of the light ray and the normal to the surface. Use the formula given above to calculate the intensity of light needed to illuminate a billboard with 600 foot-candles at a distance of 50 feet if $\theta = 15^0$. $E = \frac{I \cos \theta}{r^2}$ $I = \frac{E r^2}{\cos \theta}$ 1,666.600 lumens
E. General Identities	E. Given ten problems which use the general identities in formulating their solution, determine with 80% accuracy the solution to each problem.	
F. Conversions of Sums and Products	F. Given five products of circular functions and five sums of circular functions, convert with 80% accuracy each product to a sum and each sum to a product.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

D. Demonstrate the use of general reduction formulas. Stress that these formulas may be used to express the circular functions of a given number in terms of functions of a number between 0 and $\pi/2$.

1. $\sin(2k\cdot\frac{\pi}{2} + \beta) \equiv (-1)^k \sin \beta$
2. $\cos(2k\cdot\frac{\pi}{2} + \beta) \equiv (-1)^k \cos \beta$

E. After a class presentation and discussion, request several students to demonstrate the use of the general identities at the board.

1. $\sin 2\alpha \equiv 2\sin \alpha \cos \alpha$
2. $\cos 2\alpha \equiv 2\cos^2 \alpha - 1$
3. $\tan 2\alpha \equiv \pm \frac{-2 \tan \alpha}{1 - \tan^2 \alpha}$
4. $\sin \frac{\theta}{2} \equiv \pm \frac{1 + \cos \theta}{2}$
5. $\cos \frac{\theta}{2} \equiv \pm \frac{1 + \cos \theta}{2}$
6. $\tan \frac{\theta}{2} \equiv \pm \frac{1 - \cos \theta}{\sin \theta}$
7. $\tan \frac{\theta}{2} \equiv \pm \frac{\sin}{1 + \cos \theta}$

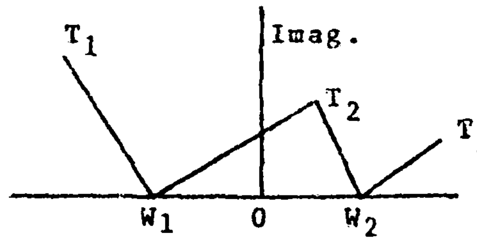
F. After a class presentation allow the students to practice converting a product of two circular functions into a sum of two functions, and vice versa. Ask several students to work at the board.

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
G. Circular Functions	G. Given ten circular function equations, find with 80% accuracy the value of θ between 0 and 2π which satifies each equation.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
G. Demonstrate the solution of circular functions for θ between 0 and 2π at the board. Allow at least one or two students to solve a circular function at the board.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
IV. Complex Numbers A. Definition	IV. THE STUDENT SHOULD BE ABLE TO: A. Define a complex number as any expression written $x + yi$, where x and $y \in R$, and i has the property that $i^2 = -1$.	Concept Relationship of complex numbers to the career of a petroleum engineer. Performance Objective Determine the position of a hidden gas well (one out of three) which is located midway between the centers of three storage tanks connected to the wells by pipes which form right isosceles triangles.
B. Modulus	B. Given ten complex numbers, calcutate with 80% accuracy the modulus in each case by using the formula $\|Z\| = \sqrt{x^2+y^2}$.	General Information Students wishing extra activities should be encouraged to research other careers in the petroleum industry. They may wish to compile and analyze material on work done by the petroleum worker.
C. Conjugates	C. Define conjugates as complex numbers which differ only in the sign of their imaginary parts.	Examples: 1. Petroleum geologist 2. Drillers 3. Shooters 4. Rig Builders 5. Derrickmen 6. Enginemen 7. Roustabouts 8. Pumpers 9. Gasoline plant engineer 10. Well puller
D. Operations	D. Solve with 80% accuracy ten problems involving operations with complex numbers.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
IV. A. Solve and explain the equation $a+1=0$ over the set of all positive real numbers. Extend the domain to R and ask the class for the solution set. Present the equation $a^2+1=0$ over R. Question: Is there any number that will satisfy such an equation? At this point the teacher should introduce imaginary numbers and define the complex number $(a+bi)$. Illustrate why $i^2=-1$. B. Define the modulus of a complex number Z, as $\|Z\|=\|x + yi\|=\sqrt{x^2+y^2}$. After several examples have been presented to the class make a writter assignment. C. Define the conjugates of two complex numbers which differ only in the sign of their imaginary parts. Write complex numbers on the board. Ask students to state the conjugate of the numbers. D. Write problems on the board in which students must perform operations in order that the complex number may be simplified. Ask students one at a time to work a problem at the board without any previous instruction. As problems arise instruct students on what must be done to alleviate the problem.	Harlandale Audio-Visual Center Career Engineering and Types of Engineering; magnetic tape - - MT - 295 For Additional Information on Petroleum Engineering American Petroleum Institute, 1271 Avenue of the Americas, New York, New York 10020 Society of Petroleum Engineers of AIME, 6200 North Central Expressway, Dallas, Texas 75206 National Petroleum Refiners Association, 1725 DeSales St. N.W., Washington, D.C. 20036 American Association of Petroleum Geologists, P.O. Box 979, Tulsa, Oklahoma 74101	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		In 1970 approximately 266,800 persons were employed by the petroleum industry. High school students wishing a career in the petroleum industry should seek guidance from a rescurce person employed by the industry for educational requirements. Teaching Activity A petroleum engineer was assigned to reopen an abandoned 3 well gas field. He found two exposed caped gas wells but the third well had become buried and unknown in location. His only notes showed the well to be located midway between the centers of two of three storage tanks connected to the wells by pipes which form right isosceles triangles as shown in the following: The length of the pipes and locations of the tanks was unknown. He must locate the third well. How? 

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		He sets up a real axis using complex numbers through W_1 and W_2 with the centers of W_1W_2 as the origin. The imaginary axis would then be perpendicular to $\overline{W_1W_2}$ through the origin. If $OW_2=1$ then $OW_1=-1$. Let tank T_2 have the complex number location $T_2 = a + bi$, then vector $W_2T_3=(a + bi)-1$. Hence, the vector $W_2T_3= -i\{(a + bi)-1\}$ produces a 90^0 clockwise rotation caused by multiplying by $-i$. Likwise, $\overrightarrow{W_1T_2}=\cdot (a + bi)-(-1)$, so that $\overrightarrow{W_1T_1}=i(a + bi + 1)$ produces a 90^0 counter clockwise rotation caused by multiplying by i. We then find $\overrightarrow{W_1T_1} + \overrightarrow{W_2T_3}=(-ia - bi^2 + i)+(ia + bi^2 + i)=2i$ with the third well located at $1/2\cdot(2i)$ or i which is on the perpendicular bisector of $\overline{W_1W_2}$ a distance of OW_2 from 0. It should also be noted that the electrical engineer often uses complex numbers (with j representing i) in his study of alternating current.

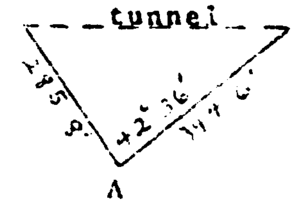
SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
V. Applications of Circular Functions to Angles	V. THE STUDENT SHOULD BE ABLE TO:	<u>Concept</u> Relationship of the law of cosines to a tunnel through a mountain being designed by a civil engineer. <u>Performance Objective</u> Given a distinct point with its distance from each end of a proposed tunnel and the angle of separation between two imaginary lines from the point, calculate the length of the tunnel. <u>General Information</u> Students wishing extra activities should be encouraged to research other careers in civil engineering. Approximately 185,000 civil engineers were employed in the United States in 1970. They were employed by public utilities, railroads, educational institutions, iron and steel industry, and other major manufacturing industries. The employment outlook for the civil engineer is for continued growth through the 1970's. The average starting salary for the civil engineer in 1970 was $10,000. With exper-
A. Angles and Arcs	A. Given the measure of ten angles in degree measure, convert with 80% accuracy to radian measure.	
B. Circular Functions of Angles	B. Given five angles in degree measure and five angles in radian measure, determine with 80% accuracy the sine and cosine of each.	
C. Geometric Use of Angles	C. Given the coordinates of ten points at which terminal sides of ten angles pass through, sketch and find with 80% accuracy the circular functions of each angle.	
D. DeMoivre's Theorem	D. Given ten expressions in the form $\boxed{r}(\cos\theta + i\sin\theta)\boxed{\,}r$, change the expressions to the form x+yi with 80% accuracy by use of DeMoivre's Theoren.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
V.	Harlandale Audio-Visual Center	
A. Review such terms as initial side, terminal side, positive angle, and negative angle. Discuss calculating the magnitude of an angle by use of the following relations: angle in revolutions $= \frac{S}{2\pi r}$ angle in degrees = (number of revolutions)$\cdot$ (360^0) angle in radians = (number of revolutions)$\cdot$ (2π) $\frac{\text{angle in degrees}}{360^0} = \frac{\text{angle in radians}}{2\pi}$ Ask several students to work problems at the board.	Career Engineering: Something More Than Talk; 16mm film -- 16-825 Civil Engineer; cassette tape -- Cas.T - 54 For Additional Information on Civil Engineering American Society of Civil Engineers, 345 East 47th Street, New York, New York 10017	
B. Discuss using a trigonometric table. After instruction orally ask selected students for the sine and cosine of a given angle. Make a written exercise on the use of the table in regard to interpolated values.	Engineers' Council for Professional Development, 345 East 47th Street, New York, New York 10017	
C. Define functions of angles in regard to the unit circle. Introduce polar coordinates and discuss the relationship between rectangular and polar coordinates.		
D. Explain to the class the derivation of DeMoivre's Theorem. Request several students to go to the board to demonstrate the use of DeMoivre's Theorem. Make a written assignment.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
E. Triangles	E. Given a listing of ten criteria for determination of a triangle, choose the four which are correct.	ience and training the salary rose to $25,600 or more. High school students wishing to enter civil engineering must have a strong background in math and science. A bachelor's degree in civil engineering is required for entry into civil engineering positions. Because of rapidly changing technology, an engineer must be willing to continue his education throughout his career.
F. Oblique Triangles 1. Law of Sines 2. Law of Cosines 3. Applications	F, 1. Given ten triangles with the necessary parts for determination of a triangle, find with 80% accuracy a designated missing part by using the law of sines. 2. Given ten triangles with the necessary parts for determination of a triangle, find with 80% accuracy a designated missing part by using the law of cosines. 3. Correctly calculate with 80% accuracy the solutions of problems involving applications involving oblique triangles.	Teaching Activity Designing a tunnel through a mountain presents a difficult task for the civil engineer. He does not have all the usual visual guidelines to help him guide the construction in the right direction. Thus, if the digging starts from both sides of the mountain, the engineer must be sure that the two parts of the tunnel meet in exactly the same place. In the illustration below the engineer is standing atop point A. One end of the tunnel is 285.8 feet from A,

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
E. Review the criteria for determination of a triangle. 1) two angles and one side are given, 2) two sides and the included angle are given, 3) three sides are given if the longest side is less than the sum of the other two. Also, there are at most two triangles when 4) two sides and an angle opposite one of them is given. Ask the students to give examples of criteria which would not necessarily determine a triangle. F. 1. Discuss using the law of sines to find a missing part of a triangle when three parts of the triangle are known. Also give examples of finding two unknown sides in a right triangle when one angle and one side are known. 2. Relate the Pythagorean Theorem to the law of cosines. Discuss using the law of cosines to find a side of a triangle when two sides and the angle opposite the unknown side are given. 3. Use the career teaching activity in this section for an application of oblique triangles. Also stress the use of oblique triangles by the navigator.		

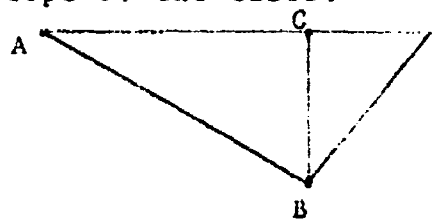
CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	<u>CAREER</u> CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		while the other end is 394.6 feet from A. The angle of separation between the two ends at A was found to be $42^0 36'$. Using this data, calculate the length of the tunnel.  Solution: Let x=length of tunnel by the Law of Cosines, $x^2=(285.8)^2+(394.6)^2-2(285.8)(394.6)\cos 42^0 36'$ x=267.1 miles

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
VI. Inverse Circular Functions A. Arc Sin θ, Arc Cos θ, and Arc Tan θ	VI. THE STUDENT SHOULD BE ABLE TO: A. Given ten circular functions, solve each problem with 80% accuracy for θ by using inverse functions (arc sin θ, arc cos θ, or arc tan θ).	Concept Relationship of the use of inverse circular functions to the ocean bottom being mapped by an oceanographer. Performance Objective Given the distance of each of two ships from a point on the ocean bottom and that the triangle formed between the three positions is a right triangle, calculate the slope of the sea bottom.
B. Operations	B. Given ten problems involving operations with inverse circular functions, solve each problem with 80% accuracy without the use of tables.	General Information Students wishing extra activities should be encouraged to research other careers in oceanography. They may wish to compile and analyze material done by the oceanographer. The minimum educational requirement for beginning professional positions in oceanography is the bachelor's degree with a major in oceanography, biology, a geoscience, one of the other basic sciences, mathematics, or engineering. In 1970 oceanographers having a bachelor's degree started working at an average salary of $8,471 per year. Advancement in salary is dependent on advanced degrees and experience.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
VI. A. Discuss the meaning of inverse functions. Note that direct trigonometric functions are single-values, while inverse trigonometric functions are many-valued. Students should become accustomed to both notations: $y=\sin^{-1}x$ and $y=\arcsin x$. However, care should be taken not to confuse the -1 with an exponent. The student should be able to recognize the graphs of the inverse functions on sight. B. Exercises are an important aid to understanding of inverse functions. Select some exercises from each kind provided by the text. Require students to sketch some graphs of inverse functions.	Harlandale Audio-Visual Center Career Oceanographers; magnetic tape -- MT - 305 For Additional Information on Oceanographic Careers American Society of Limnology and Oceanography, W.K. Kellogg Biological Station, Hickory Corners, Michigan 49060 International Oceanographic Foundation, 1 Rickenbacker Causeway, Virginia Key, Miami, Fla. 33149 National Oceanography Association, 1900 L. St. N.W., Washington, D.C. 20036 National Oceanic and Atmospheric Administration, Room 218, Bldg. 5, 6010 Executive Blvd., Rockville, Maryland 20852	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	<u>CAREER</u> CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		Although oceanography is and will remain to be a relatively small profession, the outlook is for very rapid growth in this profession through the 1970's. As a result new oceanography graduates should continue to have favorable employment opportunities. <u>Teaching Activity</u> The oceanographer often finds it necessary to make use of trigonometry. In mapping the shape of the ocean bottom for example, it is necessary to measure distances and angles of a surface hundreds or thounsands of feet under water. Suppose a research vessel is making an attempt to trace the face of a sea cliff face with sonar waves. A beam of sonar waves (AB) aimed at the cliff face is reflected in the direction BC to be 3854 and 2998 feet respectively. The oceanographer must determine the slope of the cliff.



SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		$\alpha\beta = \text{arc cos } \frac{2998}{3854}$ $\alpha\beta = \text{arc cos } .7778$ $\alpha\beta = 38^0\ 56'$ Therefore, angle between sea level and cliff face = $19^0\ 28'$.

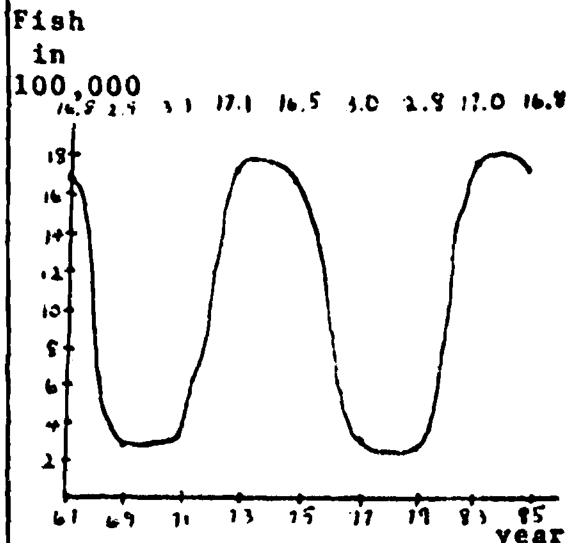
SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
VII. Graphs of Functions	VII. THE STUDENT SHOULD BE ABLE TO:	<u>Concept</u> Relationship of graphing periodic functions to a study made by a biologist of perch population. <u>Performance Objection</u> Given a sinusoidal curve of the perch population in Lake Michigan, predict the population in 1984. <u>General Information</u> Students wishing extra activities should be encouraged to research other careers in life science. They may wish to compile and analyze material on work done by the life scientist. Examples: Botanists Zoologists Microbiologists Agronomists Anatomists Biochemists Biological Oceanographer Biophysicists Ecologists Embryologists Entomologists Geneticists Horticulturists Husbandry Nutritionists Pathologists Pharmacologists Physiologists
A. Periodic Functions	A.	
1. Amplitude	1. Define the amplitude of a function as the greatest ordinate a, or maximum of a function.	
2. Period	2. Define the period of a function as the length of one complete cycle, $2\pi/k$.	
3. Phase Shift	3. Define phase shift as the amout the graph of a function is moved to the right or left or the y-axis.	
B. $y = a \sin kx$	B. Given ten functions of the form $y = a \sin kx$, find the amplitude, period, and sketch the graph of each with 80% accuracy.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
VII. A. 1. If possible, bring several graphs to class (from magazines, newspapers, etc.)which illustrate a periodic function. Discuss the purpose of the graph and its application to a future career. Stress that the highest point illustrated on the graph is considered the amplitude of the function. 2. The career teaching activity in this section may prove helpful in discussing the period of a function. Emphasize how often ecological balance may be represented by a periodic function. Application in the study of electronics and mechanical physics (simple harmonic motion, sound waves, light, etc.) 3. Stress that in the generalized sine function defined by $y=a \sin(kx + b)$ the b represents the phase shift of the function. Discuss a possible phase shift in the trigonometric curve mentioned in the career teaching activity (possibly caused by pollution or natural disaster). B. After instruction assign the students at least ten functions of the form $y=a \sin kx$. In each function the students should find the amplitude and period as well as make a sketch of the graph. Ask selected students to display their work at the board.	<u>For Additional Information on Biological Careers</u> American Institute of Biological Sciences, 3900 Wisconsin Ave. NW., Washington, D.C. 20016 American Society of Horticultural Science, 615 Elm Street, St. Joseph, Michigan 49085 American Physiological Society, 9650 Rockville Pike, Bethesda, Maryland 20014 Ecological Society of American, Connecticut College, New London, Connecticut 06320	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
C. $y = a \sin \cdot (kx + b)$	C. Given ten functions of the form $y = a \sin \cdot (kx + b)$, find the amplitude, period, phase shift, and sketch the graph of each with 80% accuracy.	Approximately 180,000 persons were employed in the life sciences in 1970. Salaries ranged from \$6,548 to more than \$26,100. Students wishing a career in life science should have obtained all available science courses with strong emphasis on math. A bachelor's degree may be adequate for some positions, but opportunities for promotion are few without graduate training. Teaching Activity The perch which inhabit our Great Lakes demonstrates a cyclic population. The biologist often studies these populations in order to predict the effect of the lamprey on other inhabitants of the environment. This kind of cyclic population change in nature is quite common and can be represented by a trigonometric function. Using the data below on the perch population in Lake Michaigan, construct a graph. Then, from the shape of this graph, try to predict the fish population for 1984.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
C. In consideration of the function $y=a \sin(kx + b)$ stress that b represants the phase shift of the function. Encourage students to determine the phase shift of the function by inspection. Assign the students at least ten functions of the form $y=a \sin(kx + b)$. The students should find the amplitude, period, phase shift, and sketch the graph of each function. Ask selected students to display and explain their work at the board.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		 This graph is a sinusoidal function with maxima near 18 for 1966 and 1974. Minimum occurs near 2.5 for 1970 and 1976. Another maximum should occur in 1984 near 18.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

AUDIO-VISUAL SOURCE INFORMATION

TITLE	TYPE	SOURCE	TIME	B/W OR COLOR
Careers in Television	record FS	Melton Book Company		C
Civil Engineer	Cas. T.	Educational Progress Corp.		
Economics in Business Series	TP	3 M		
Economist	MT	Guidance Associates		
Engineering and Types of Engineering	MT	Guidance Associates		
Engineer: Something More Than Talk	16 mm	Purdue University	27 min.	C
Introduction to Functions	FS	Popular Science		C
Oceanographers	MT	Guidance Associates		

ANALYTIC GEOMETRY
(a two quater course)

CURRICULUM GUIDE

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San Antonio, Texas

The audio-visual materials listed in this guide have been assigned catalogue numbers by the Harlandale Independent School District audio-visual department, San Antonio, Texas.

A C K N O W L E D G E M E N T S

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Mr. Charles Snyder

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Mr. William R. Marshall - Associate Superintendent

Mr. William H. Bentley - Director of Vocational Education

Miss Mary E. Daunoy - Secondary Consultant

Mr. Hamilton C. Dupont (deceased) - Head of Math Department

Gratitude is also expressed to the Texas Education Agency, Minnie Stevens Piper Foundation, Audio-Visual Staff, and the Curriculum Staff.

DNS:ARE

Preface

Meaningful existence is the goal of life in today's world. Living takes on meaning when it produces a sense of self-satisfaction. The primary task of education must be to provide each individual with skills necessary to reach his goal.

When children enter school, they bring with them natural inquisitiveness concerning the world around them. Normal curiosity can be the nucleus which links reality to formal training if it is properly developed. A sense of continuity must be established which places education in the correct perspective. Communities must become classrooms and teachers resource persons. Skills such as listening, problem solving, following directions, independent thinking and rational judgement then can merge into daily living procedures.

In classrooms especially designed to form a bridge between school and the world of work, experiences must be developed. On campus performance in job tasks and skills, following a planned sequence of onsite visitation, will fuse information into reality. Practical relationships developed with those outside the formal school setting will provide an invaluable carry-over of learned skills.

Search for a rewarding life vocation is never easy. Without preparation it becomes a game of chance. With a deliberate, sequential, and planned program of development, decisions can be made based upon informed and educated judgements.

A full range career education program, K-12, will offer opportunities for participants to enter employment immediately upon completion of training, post secondary vocational-technical education, and/or a four-year college career preparatory program.

C. N. Boggess

C. N. Boggess, Superintendent
Harlandale Independent School District

The Career Education Project has been conducted in compliance with the Civil Rights Act of 1964 and is funded by a grant from the U. S. Office of Education and the Texas Education Agency.

P H I L O S O P H Y

The educational needs of any community are somewhat unique. This was certain to have been one of the guiding principles used when our forefathers set up local control for public schools. Accordingly, the philosophy of the Harlandale school system is to serve the educational needs of all its citizens as evidenced by adult classes, government sponsored retraining programs, vocational courses, etc. The mathematics department follows this philosophy in planning a program best suited to the needs of our students.

The past decade has proven the need for the new emphasis on the importance of the study of mathematics. Usefulness of mathematics in many fields of learning and endeavor, long thought to be free of mathematics, is now an accepted reality. Also, basic principles must be understood in order that mathematical systems can be devised to describe new human or mechanical activities, as they come to pass.

As mathematics continues to grow this must, of necessity, result in the addition of new symbols, terms, topics, and new approaches. The changing times will make some older topics and methods obsolete. To meet the mathematical needs of our students and to assist teachers in their instruction, the mathematics department has prepared this mathematics guide. Any given faculty consists of personnel with different degrees of training, experience, local tenor, and understanding of student needs. Hence, the desirability for some guiding criteria. In addition, we feel all students should consider career planning as a major facet of their education. Thus it follows that they will need some exposure to the different mathematical requirements of the varied career fields. In part, it is the purpose of this guide to assist the teacher in providing appropriate instruction to meet such needs.

The department feels that the most important basic guide for any mathematics course is the textbook, and careful care is taken in the selection of this book. Not only is the textbook an important guide for the teacher but it is also desirable for the student to learn the use of a textbook as a guide and important tool for learning.

Therefore, the plan of this mathematics guide is not to rewrite the textbook but to improve on it. Generally, the plan is to implement, where desirable, the textbook coverage, describe supplementary material that is needed and make suggestions on methods, procedures, order of coverage, etc.

Mathematics is a thoughtful, creative and intellectually stimulating subject. The enthusiasm and interest of the teacher in the subject is the best atmosphere for creating student enthusiasm for mathematics. This guide is planned to help foster this enthusiasm and in no way infringes on the academic freedom of the teacher.

It is hoped that the guide will prove helpful in understanding the basic standards, improving instruction, and developing the desired uniformity for the classes in each area of study. Finally, the guide should serve as the nucleus for a continuing effort to improve mathematics instruction.

Mr. Hamilton C. Dupont (deceased) - Head of Math Department
Harlandale Independent School District

ANALYTIC GEOMETRY

Only those students who have done well in trigonometry should be advised to take analytic geometry. It is a course which requires considerable mathematical maturity and dedication on the part of the student.

Goals

To give the student with talent in mathematics additional mathematical experiences in a subject area which will broaden his perspectives

To give the college-bound student, who will not be taking the high school calculus course, additional preparation for any college calculus course

To give additional mathematical training which will enhance the student's understanding of other courses he may take in high school or college such as physics, engineering, etc.

To improve the student's understanding of the relationship between various careers and analytical geometry concepts

ANALYTICAL GEOMETRY

The following outline is built upon a "conceptual ladder" (concepts from 'easiest to hardest') for analytical geometry. The outline corresponds to the outline found in the curriculum concepts of this guide. The page numbers refer to the present textbook being used in analytical geometry by the Harlandale Independent School District. (Fuller, Gordon, Analytic Geometry. Reading, Massachusetts: Addison - Wesley, 1964.)

I. The Coordinate System
   - A. Distance Between Two Points pp. 5-9
   - B. Slope of a Line pp. 9-12
   - C. Angle Between Two Lines pp. 12-16
   - D. Division of a Line Segment pp. 16-18
   - E. Equation of a Locus pp. 24-27

II. Lines and Circles
   - A. First Degree Equation and Its Locus pp. 28-35
   - B. Distance from a Point to a Line pp. 35-40
   - C. Families of Lines pp. 38-40
   - D. Equations of Circles pp. 43-49
     - 1. Center - Radius Form
     - 2. General Conic Form
     - 3. Determination of Circles by Geometric Conditions
       - a. Three Points That Determine a Circle
       - b. Circle Tangent to a Line
       - c. Circle Tangent to Two Lines
   - E. Translation of Axes pp. 49-52

III. Conics
   - A. Parabola pp. 54-63
     - 1. Simple Form
       - a. Major Axis
       - b. Focus
       - c. Directrix
       - d. Latus Rectum
     - 2. Vertex at (h,k)
     - 3. Symmetry
   - B. Ellipse pp. 63-73
     - 1. Simple Form
       - a. Major and Minor Axes
       - b. Foci
       - c. Latus Rectum
     - 2. Center at (h,k)

- C. Hyperbola pp. 73-78
  - 1. Simple Form
    - a. Transverse and Conjugate Axes
    - b. Foci
    - c. Latus Rectum
    - d. Asymptotes
  - 2. Center at (h,k)

IV. Simplification of Equations
- A. Translation pp. 82-85
  - 1. Given Origin
  - 2. Finding New Origin
- B. Rotation of Axes pp. 85-88
  - 1. Given Angle
  - 2. Finding Proper Angle
- C. Combining Rotation and Translation pp. 88-92
- D. Identification of a Conic pp. 92-93
- E. Graphing by Addition of Ordinates pp. 93-96

V. Algebraic Curves - Analysis and Graphing pp. 97-106
- A. Intercepts
- B. Asymptotes
  - 1. Horizontal - Vertical
  - 2. Slant

VI. Transcendental Curves
- A. Trigonometric Equations pp. 109-114
  - 1. Simple Functions
  - 2. Inverse Functions
- B. Exponential Curves p. 114
- C. Logarithmic Curves pp. 114-116
- D. Graphing by Addition of Ordinates pp. 116-117

VII. Polar Coordinates
- A. Polar Coordinate System pp. 118-120
- B. Relations between Polar Coordinates and Rectangular Coordinates pp. 121-124
- C. Graphs of Polar Coordinate Equations pp. 124-129
- D. Special Polar Coordinate Equations pp. 130-133

VIII. Parametric Equations
- A. Definition p. 142
- B. Equations and Graphs
  - 1. Circle and Ellipse pp. 143-148
  - 2. Path of a Projectile pp. 148-150
  - 3. Cycloid pp. 150-153

IX. Vector Approach to Analytic Geometry

- A. Review of Rectangular Coordinates pp. 2-27
  1. Equations
  2. Vectors
- B. Scalar Components of Vectors pp. 183-185
- C. Direction Cosines pp. 203-204
- D. Angle between Vectors and Dot Product pp. 204-208
- E. Parallel and Perpendicular Vectors
- F. Dividing Line Segments pp. 16-18
- G. Complementary Vectors
- H. Area of Triangles and Bar Products
- I. Equation of a Line by Use of Dot Product pp. 199-202
- J. Vectors in Space pp. 188-190
- K. Equation of a Plane in Space pp. 195-199

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
I. The Coordinate System A. Distance between Two Point	I. THE STUDENT SHOULD BE ABLE TO: A. Calculate with 80% accuracy the distance between two pairs of points on a coordinate plane by using the formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.	Concept Relationship of the rectangular coordinate system to the job of a surveyor. Performance Objective Given the coordinates of two points or cities, determine the distance between the cities by use of the distance formula. General Information Students wishing extra activities should be encouraged to research other careers in surveying. They may wish to compile and analyze material on work done by surveyors.
B. Slope of a Line	B. Given ten lines on a coordinate plane, determine their slope with 80% accuracy by using the formula $m=\frac{y_2-y_1}{x_2-x_1}$.	Surveying is a limited occupation in regard to the number of jobs available.
C. Angle between Two Lines	C. Given the slope of ten pairs of intersecting lines, determine the angle of intersection of any pair with 80% accuracy by using the formula $\tan\phi=\frac{m_2-m_1}{1+m_1m_2}$.	There are approximately 50,000 licensed surveyors in the United States. The average salary for surveyors in the United States in 1970 was $8,850 per year.
D. Division of a Line Segment	D. Given ten line segments on a coordinate plane, calculate with 80% accuracy the mid-point of each line segment by using the formulas $x=x_1+r(x_2-x_1)$ and $y=y_1+r(y_2-y_1)$.	The most common method of preparing for work as a surveyo[r] is through a combination of post-secondary school courses in surveying and extensive on-the-job training in survey techniques and in the use of survey instruments.
E. Equation of a Locus	E. Define on a written exercise the equation of a locus as a relation between x and y which is satisfied by the coordinates of all points of the locus and by no others.	The entranc[e] requirement for most surveying programs is high school graduation (preferably includin[g] courses in algebra, geometry,

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
I. A. Briefly review the rectangular coordinate system. Ask a student to give the coordinates of two points. Calculate the distance between the two points by using the formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$. Ask several students to repeat the process at the board. B. Define a linear function (using the form ax + by=c) as a linear equation in 2 variables. Illustrate the slope of a line. Make a chart to illustrate how the slope determines the direction of a line (using positive and negative slopes). C. Draw a set of intersecting lines on a coordinate system. Write the formula tan ϕ between the two lines. Select a student to go to the board and try to find angle ϕ without further instruction. Give the student help if necessary. D. Draw a line segment on a coordinate system for which the student may visually determine the midpoint. Then introduce the formulas for determining the midpoints. Use the formulas on the line segment which has been visually determined. E. Present the derivation of the equation of a straight line, a parabola, and a circle. Ask several students to repeat the procedure at the board.	<u>Career</u> Harlandale Audio-Visual Center <u>Your Future as a Surveyor</u>; magnetic tape -- MT - 270 <u>For Additional Information on Surveying Careers</u> American Congress on Surveying and Mapping Woodward Building 733 15th St. NW. Washington, D. C. 22046 . American Society of Civil Engineers 345 East 47th Street New York, New York 10017. American Society of Photogrammetry 105 North Virginia Ave. Falls Church, Va. 22046.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	<u>CAREER</u> CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		trigonometry, calculus, and mechanical drawing). <u>Teaching Activity</u> A surveyor must often determine the distance between two points or cities on a map. He may accomplish his purpose by using rectangular coordinates determined by lines of latitude and longitude. The rectangular coordinates of a point uniquely define its position with respect to a known horizontal datum. The distance D between the two points is given by the following equation: $D=\sqrt{(Y_2-Y_1)^2+(X_2-X_1)^2}$ <u>Example</u>: The distance is to be found between the towns of Richmond and Kenney. The coordinates of Richmond are Y_2= 28,221 miles and X_2= 20,370 miles. The coordinates of Kenney are Y_1= 18,972 miles and X_1= 17,146 miles. <u>Using the distance formula</u>: $D=\sqrt{(28,221 - 18,972)^2+(20,370 - 18,972)^2}$ $D=\sqrt{(9,249)^2+(3,224)^2}$ $D=\sqrt{29,818,776 + 10,394,176}$ $D=\sqrt{40,212,952}$: D=6,341 miles

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
II. Lines and Circles A. First Degree Equation and Its Locus	II. THE STUDENT SHOULD BE ABLE TO: A. Write the locus of a first-degree equation as $Ax + By + C = 0$. Given two points on each of ten lines, determine with 80% accuracy the equation of the line.	Concept Relationship of finding the distance between a point and a line to the job of the draftsman. Performance Objective Given the equation of a line representing the base of a gasket and the coordinates of a point which is to be placed and drilled on the gasket, calculate the perpendicular distance from the point to the line. General Information Students wishing extra activities should be encouraged to research drafting as a career. They may wish to compile and analyze material on work done in drafting. Approximately 310,000 draftsmen were employed in 1970; almost 4 percent were women. High school students wishing to enter drafting can obtain training from a number of sources (technical institutions, junior and community colleges, vocational and technical high schools, correspondence schools and 3 or 4 year apprenticeship programs). The average salary for a draftsman was $700 per month in 1970. Employment opportunities for draftsmen ar
B. Distance from a Point to a Line	B. Given ten lines and ten points not on the lines, calculate with 80% accuracy the distance from a specified point to a specified line by using the formula $d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$.	
C. Families of Lines	C. Define a family of lines as a collection of lines defined by a linear equation with one parameter.	
D. Equations of Circles 1. Center-Radius Form	D. 1. Given ten circles with the abscissas and ordinates of the center and a point on the circle, calculate with 80% accuracy the length of the radius of each circle.	
2. General Conic Form	2. Given ten general conic form equations of circles, determine with 80% accuracy the center of each circle by completing the square.	
3. Determin-tion of Circles by Geometric Conditions	3. Given three points on each of ten circles, determine with 80% accuracy the equation of each circle.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
II.		
A. Present and discuss each form of straight line equations. Ask selected students to work and explain examples in front of the class. (forms: general form, slope-intercept form, point-slope form, two-point form, and intercept form)	Career Harlandale Audio-Visual Center *Your Future as a Draftsman*; magnetic tape -- MT-258	
B. Discuss and develop the method for finding the distance from a point to a line. Apply this method to finding the distance between two parallel lines.	For Additional Information on Drafting Careers	
C. Discuss families of lines by use of the various forms of the equation of a line. Present the families of lines passing through the interesection of two lines. Graph several families of lines on a coordinate system.	American Federation of Technical Engineers 1126 16th Street NW., Washington, D.C. 20036	
D. 1. Define and discuss the characteristics of circles. Develop the equations of a circle, first with its center at the origin and second with its center at point (h,k). 2. Present the derivation of the general conic form from the standard form. After instruction, give the students several equations in general form and ask them to determine the locus of each. 3. Discuss each of the geometric condition of a circle. Make a written assignment which concerns each of the conditions. a. Three Points That Determine a Circle b. Circle Tangent to a Line	Amerıcan Institute for Design and Drafting Post Office Box 2955 Tulsa, Oklahoma 74101	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
E. Translation of Axes	E. Given ten problems, calculate with 80% accuracy the new coordinates of the point $P(x,y)$ if the origin is moved to (x^1, y^1).	expected to be favorable through the 1970's. Prospects will be best for those having post-high school drafting training. Teaching Activity A draftsman is a person who draws plans and blueprints of machines, houses, automobiles and almost anything that can be manufactured. These plans must be precise in their dimensions, and are usually drawn to scale. Suppose you are a draftsman and are working for a company which makes parts for automobile engines. A blueprint for a gasket is needed which has a base line represented by the equation $15x - 8y - 51 = 0$ and a hole represented by the point (10,3). In making the drawing it is necessary to determine the distance from the line to the point. To find this distance one uses the formula: $d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$ $d = \frac{15(10) - 8(3) - 51}{\pm\sqrt{(15)^2 + (-8)^2}}$ $d = \frac{150 - 24 - 51}{17}$ $d = \frac{75}{17} = 4\frac{7}{17}$ units

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
c. Circle Tangent to Two Lines E. Draw a circle on a coordinate system with its center not at the origin. Ask a student to give the coordinates of the center of the circle and name the distance of abscissa and ordinate from the original axes. After this discussion present the translation of axes for the circle.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
III. Conics	III. THE STUDENT SHOULD BE ABLE TO:	<u>Concept</u> Relationship of a parabola to a cable supported bridge being designed by an architect. <u>Performance Objective</u> Correctly calculate the height of the towers supporting a bridge when given the quadrati equation of the supporting parobolic cables and the linea distance between the two supporting towers. <u>General Informtion</u> Students wishing extra activities should be encouraged to research other careers in architecture. They may wish to compile and analyze material o specific architectural careers Examples: 1. Architect, construction 2. Architect, landscaping (salaries range from \$7,000 to over \$25,000 a year) A solid background in math is necessary for the high school student wishing to go into architecture. Requirements for an architect generally require graduation from an accredited professional school followed by 3 years of practical experi ence in an architect's office. A license (acquired by test) : required by all states.
A. Parabola 1. Simple Form a. Major Axis b. Focus c. Directrix d. Latus Rectum 2. Vertex at (h,k) 3. Symmetry	A. 1. Given a drawing of a parabola, label the major axis, focus, directrix, and latus rectum. 2. Given the equations of ten parabolas in the form $(y-k)^2=4a(x-h)$ or $(x-h)^2=4a(y-k)$, determine by the inspection of <u>a</u> in which direction the parabola opens. 3. Define a curve as being symmetric with respect to a line if each of its points is one of a pair of points with respect to the line.	
B. Ellipse 1. Simple Form a. Major and Minor Axes b. Foci c. Latus Rectum 2. Center at (h,k)	B. 1. Given a drawing of an ellipse, label the major axis, minor axis, foci, and latus rectum. 2. Given the foci and verticies of ten ellipses, determine with 80% accuracy the equation of each ellipse.	
C. Hyperbola 1. Simple Form a. Transverse and Conjugate Axes	C. 1. Given a drawing of a hyperbola label the transverse axis, conjugate axis, foci, latus rectum, and asymptotes.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
III. Discuss conics in general. Present the figures derived by dissecting a cone. Use a dissectable cone to illustrate these principles. A. Define and develop the equation for a parabola. Present its characteristics and principle parts. Develop equations for these parts. Present the simple form first, then the form in which the vertex is at a point (h,k). Expand the equation to the general conic form of the equation. Discuss the symmetry and a simple method of graphing. Discuss the equation of a parabola given certain geometric conditions. Discuss the use of the parabola in parabolic reflectors, paths of thrown objects, ect. B. Define and develop the equation of an ellipse. Discuss its characteristics and how to find its dimensions from its equation. Develop the equation for the ellipse with its center at a point (h,k). Expand the equation to the general conic form. Discuss finding the equation of an ellipse from given geometric conditions. Define the meaning of eccentricity and show the effect on an ellipse for different values. Given some values of the eccentricity of different orbits for different planets of the solar system. A comparison of the values of Mercury (0.206) and the earth (.017) are especially important. C. Define and develop the equation of a hyperbola. Discuss its characteristics and how to find its dimensions from the equation. Discuss asymptotes and stress that of the conic sections only the hyperbola	<u>Curriculum</u> Harlandale Audio-Visual Center <u>Geometry in Art</u>; filmstrip -- X-38 <u>Career</u> Harlandale Audio-Visual Center <u>Mathematics in Architecture Series</u>, color slides (22 in series) -- CS-1 <u>For Additional Information on Architectural Careers</u> National Architectural Accrediting Board 521 Eighteenth Street, N.W. Washington, D.C. 20006. Society of American Registered Architects 1821 Jefferson Place, N.W. Washington, D.C. 20036 The American Institute of Architects 1785 Massachusetts Ave., N.W. Washington, D.C. 20036	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
b. Foci c. Latus Rectum d. Asymptotes 2. Center at (h,k)	2. Given the equations ten hyperbolas, sketch their curve with 80% accuracy.	<u>Teaching Activity</u> In designing a bridge supported by cables an architect knows that each cable is related to the quadratic equation $y = \frac{97}{882,000} x^2$. He also knows that the towers which support the cables must be 4,200 feet apart. His problem is to calculate the height of the supporting towers. In this case y represents the height of the cable above the midpoint of the cable and x represents the horizontal distance from the midpoint. The architect determines that the towers will be 2,100 ft. (horizontal distance) from the midpoint of the cable. Substituting this value he obtains $y = \frac{97}{882,000} (2,100)^2$ $y = \frac{97}{882,000} (4,410,000)$ $y = 97 \cdot 5 = 485$ feet tall.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
has asymptotes. Demonstrate how asymptotes aid in graphing the hyperbola. Develop the equation of a hyperbola with its center at a point (h,k). Expand the equation to the general conic form. Discuss finding the equation of a hyperbola from given geometric conditions.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
IV. Simplification of Equations A. Translation 1. Given Origin 2. Finding New Origin	IV. THE STUDENT SHOULD BE ABLE TO: A. 1. Given ten equations, determine with 80% accuracy a new equation for each if the origin is translated to the given point. 2. Given ten equations, determine with 80% accuracy the point to which the origin must be translated in order that the transformed equation shall have no first-degree term.	Concept Relationship of translation of axes to air circulation being studied by a dynamic meteorologist. Performance Objective Name one way the dynamic meteorologist uses translation of axes. General Information Students wishing extra activities should be encouraged to research other careers in meteorology. They may wish to compile and analyze material on work done by the meteorologist.
B. Rotation of Axes 1. Given Angle 2. Finding Proper Angle	B. 1. Given ten equations and an angle θ for each, determine with 80% accuracy the new equation when the axes are rotated through the given angle. 2. Given ten equations similar to $Ax^2 + Bxy + Cy^2 + Dx + Ey + F=0$, determine with 80% accuracy the angle of rotation in each problem such that the transformed, equation will have no $x'y'$- term.	A bachelor's degree with a major in meteorology is the usual minimum educational requirement for beginning meteorologists in weather forecasting. However, a bachelor's degree in a related science or in engineering is acceptable for many positions, provided the applicant has credit for courses in meteorology.
C. Combining Rotation and Translation	C. Given ten equations, simplify with 80% accuracy each problem by first using a translation of axes and then a rotation of axes.	In 1970 the average starting salary for meteorologists was \$9,000 per year. Approximately 10 percent of the meteorologist in 1970 made over \$22,300. The employment outlook for meteorlogists is expected to be favorable through the 1970's. In addition to job opportunities resulting from the rapid growth

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
IV. A. Discuss the general conic equation. Discuss simplification by the two methods of translation of axes. Review previous first to an equation with a given origin and then to the case in which the origin must be found. Present and discuss the two methods for finding the origin. B. Discuss the rotation of axes and its effect on the equation and locus. Use overlays to illustrate this operation. Discuss rotation through a given angle and draw a graph of each form of the equation. Develop the method for finding the angle of rotation required to simplify a general equation. C. Stress that in some cases a combination of rotation and translation is necessary. Give several examples.	Career Harlandale Audio-Visual Center Weather Scientists; 16 mm film -- 16-381 For Additional Information on Meteorological Careers American Meteorological Society, 45 Beacon St. Boston, Mass. 02108. American Geophysical Union, 2100 Pennsylvania Ave.,NW., Washington, D.C. 20037.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
D. Identification of a Conic	D. Given ten conic equation, determine with 80% accuracy the nature of the graph of each when it exists by determining if $B^2 - 4AC$ is less than, greater than, or equal to zero.	expected in this profession, several hundred new meteorologists will be needed each year to replace those who transfer to other fields, retire, or die.
E. Graphing by Addition of Ordinates	E. Given ten nondegenerate conic equations, sketch with 80% accuracy each graph by using the addition of ordinates method.	<u>Teaching Activity</u> Meteorologists must often consider changes in level of air circulation. A change in the level of an upper level of air circulation will trigger changes in ground level weather conditions. The meteorologist determines an equation for the upper level of circulation. When a change in level of circulation occurs, the meterologist must perform a translation of axes to study the effect of the change on other weather conditions. An exact example is not given at this time because of the calculus involved in translations of this type.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
D. & E. Discuss the descriminant used to identify the locus of a general equation. Present the simplified method of graphing by adding ordinates of two parts of the same equation.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
V. Algebraic Curves-Analysis and Graphing A. Intercepts B. Asymptotes 1. Horizontal-Vertical 2. Slant	V. THE STUDENT SHOULD BE ABLE TO: A. Given ten algebraic curves, determine with 80% accuracy the x- and y- intercepts of each equation if they exist. B. Given ten equations of the form $y = \frac{Ax^n + (\text{terms of lower degree})}{Bx^m + (\text{terms of lower degree})}$, sketch with 80% accuracy the asymptotes of each equation.	Concept Relationship of the study of algebraic curves to job of a ballistic officer. Performance Objection Given the speed of a bullet, determine the amount of time that will elapse before the bullet will reach the ground after the bullet has been fired vertically. General Information Students wishing extra activities should be encouraged to research other careers in police work. They may wish to compile and analyze material on specific police careers. Examples: 1. Guards and watchmen (approximately 200,000 were employed in 1970 with salaries ranging from \$3,848 to \$15,000 yearly) 2. FBI agents (approximately 7,900 were employed in 1970 with salaries ranging from \$10,369 to \$23,000 yearly) 3. Police officers (approximately 330,000 full-time were employed in 1970 with salaries ranging from \$8,500 for a new officer to \$23,000 for such positions as chiefs

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
V. A. Graph several algebraic curves (examples should include no intercepts, one, several or many). Stress that intercepts are of special significance in many problems. B. Note that asymptotes facilitate the graphing of an equation. Discuss equations for horizontal, vertical, and slant asymptotes. Discuss the use of symmetry for simplification of graphing.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		or commissioners). Because of specialization in police work it should be emphasized that a high school student be prepared to take college courses. A ballistic officer must have a solid background in math. FBI agents are required to have a law degree. Teaching Activity A ballistic officer must often study the trajectory of bullets. In a police case a ballistic officer is called in to determine the amount of time that elapsed from the time a gun was fired until it reached the ground. In this case the bullet was fired vertically at a speed of 1,600 feet per second. The ballistic officer knows that he can determine the elapsed time by using the formula $h = rt - 16t^2$ (h=height, r=feet per second, and t=elapsed time) Solution: The officer considers a rectangular coordinate system in which a graph of the path of the bullet is drawn. The graph will intercept the x axis in two places (where the bullet is fired and where the bullet strikes the ground). Substituting:

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

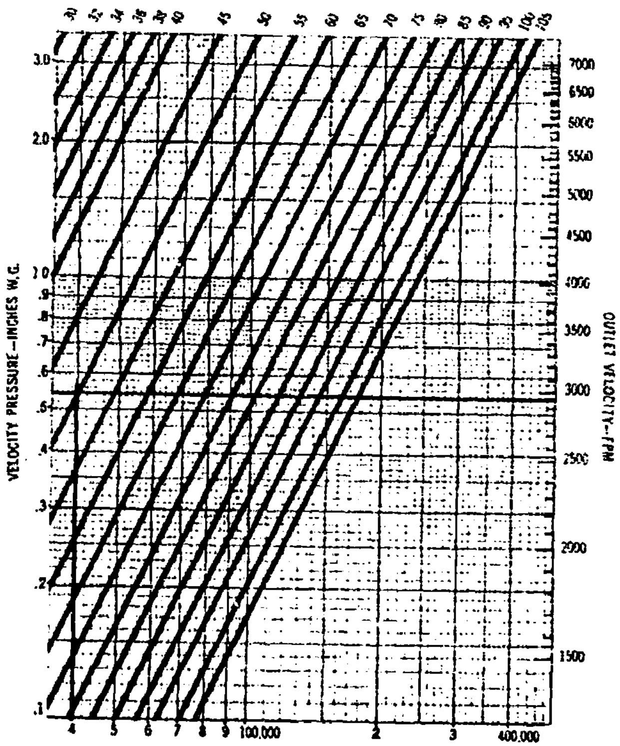
CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		$0 = 1600t - 16t^2$ $16t^2 - 1600t = 0$ $16t(t - 100) = 0$ $t = 0$ $t = 100$ seconds until the bullet reaches the ground

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
VI. Transcendental Curves A. Trigonometric Equations 1. Simple Functions 2. Inverse Functions B. Exponential Curves C. Logarithmic Curves D. Graphing by Addition of Ordinates	VI. THE STUDENT SHOULD BE ABLE TO: A. 1. Given ten trigonometric functions, sketch with 80% accuracy the graphs of sine, cosine, tangent, contangent, and secant functions. 2. Given ten inverse trigonometric functions, sketch the graph and determine the period with 80% accuracy of arc sine, arc cosine. B. Given ten exponential equations of the form $y=2^x$, $y=e^x$, or $y=10^x$. C. Given ten logarithmic equations, sketch their graph with 80% accruacy. D. Given ten transcendental equations, sketch with 80% accuracy each graph by using the addition of ordinates method.	<u>Concept</u> Relationship of the study of logarithmic curves to the job done by an air-conditioning technician. <u>Performance Objective</u> Given the CFM and the duct diameter of an air-conditioning system, determine the outlet velocity and velocity pressure of the system by using a logarithmic chart. <u>General Information</u> Students wishing extra activities should be encouraged to research other careers in the technical field. Employment opportunities for engineering and science technicians are expected to be very good through the 1970's. The demand will be strongest for graduates of post-secondary school technician training programs. Starting salaries ranged in 1970 from \$4,621 to \$8,300. In 1970 annual salaries of workers in responsible technician positions averaged almost \$11,000 and approximately one-fourth of the workers had annual salaries above \$11,900. <u>Teaching Activity</u> An air conditioning technician must often determines the outlet velocity and velocity

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
VI. A. Discuss and review the graphs of the trigonometric functions and their inverses. Stress the importance of trigonometric functions in science. B. Stress that an exponential equation is one in which a variable appears in an exponent. Present the graphs of $y=2^x$, $y=e^x$, and $y=10^x$. C. Stress that $a^y=x$ and $y=\log_a x$ express the same relation between a,x, and y. The first is the exponential form and the second is the logarithmic form. Note that the values 10 and e are the most commonly used bases for logarithms. Present the graph of $y=\log_e x$ and $y=\log_{10} x$. D. Present the students with a problem of the form $y=\sqrt{x}+\sin x$. Ask for suggestions on graphing the equation. Ask a student to graph at the board $y=\sqrt{x}$. Ask another student to graph $y=\sin x$ on the same coordinate system. Demonstrate the graph of $y=\sqrt{x}+\sin x$ by the addition of ordinate method.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		pressure of an air-conditioning system. In performing this task he most often uses a grpah which utilizes logarithmic curves. This type of graph is used for computations involving many repetitive operations dealing with the same data. For example, given that flow of 40,000 cfm (cubic feet per minute) is desired in a 50" diameter duct, we can find the velocity pressure and outlet pressure by reading directly up from the point labeled 40,000 cfm on the bottom horizontal to the diagonal labeled 50" on the top horizontal giving the duct diameter. We read to the right, and to the left from the point of intersection and find a velocity of 2940 fpm and a velocity pressure of 0.54" W.G. It should be explained to the students that this information has been plotted on a logarithmic grid which gives the logarithmic curve the appearance of a straight line.



SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

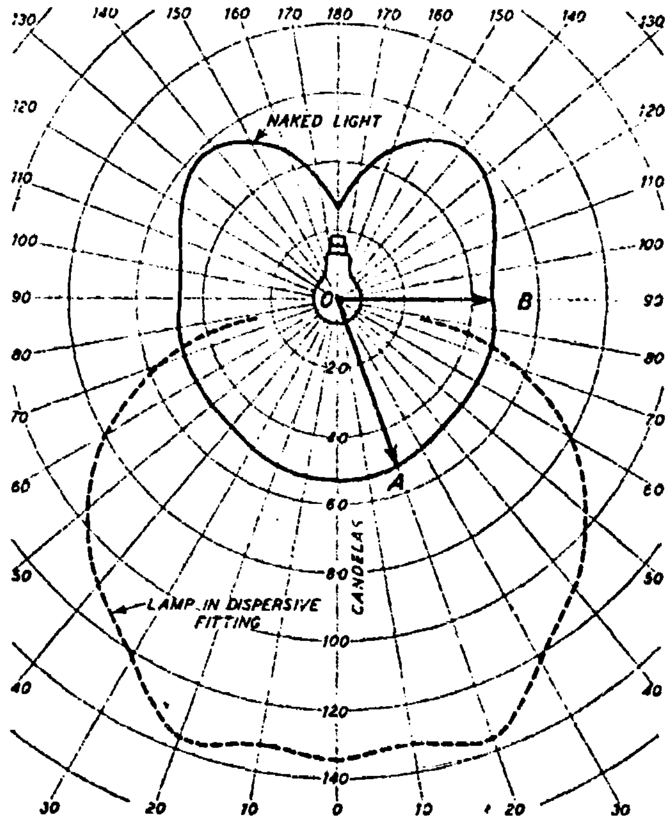
CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
VII. Polar Coordinates	VII. THE STUDENT SHOULD BE ABLE TO:	Concept Relationship of the study of polar coordinates to the job of an illustrator. Perforance Objective Given a polar curve of the candlepower of a lamp, state the candlepower of the lamp at 20^0 and 90^0 from the vertica General Information Students wishing extra activities should be encouraged to research other careers in commercial art. They may wish to compile and analyze material on work done by the commercial artist. Employment of commercial artists through the 1970's is expected to increase slowly primarily as a result of the upward trend in business expenditures for visual advertising. Artistic ability and good taste are the most important qualifications for success in commercial art, but it is essential that these qualities be developed by specialized training in the techniques of commercial and applied art. In addition, education in the fine arts, -- painting sculpture, or architecture -- and in academic studies
A. Polar Coordinate System	A. Given ten points and their polar coordinates, plot the graph of each point on a polar coordinate system with 80% accuracy.	
B. Relations between Polar Coordinates and Rectangular Coordinates	B. Given five polar coordinate equations and five rectangular coordinate equations, transform with 80% accuracy each equation to the opposite form by the use of transformation formulas.	
C. Graphs of Polar Coordinate Equations	C. Given ten polar coordinate equations, graph the equations on a polar coordinate system with 80% accuracy.	
D. Special Polar Coordinate Equations	D. Given ten "special polar coordinate equations" (rose curves, limacons, lemniscates, and spirals), graph the equations on a polar coordinate system with 80% accuracy.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
VII. A. Present and discuss the polar coordinate system to the students. Stress that the coordinates of any point on the polar coordinate system must include a distance from a fixed point and a direction from a fixed line. B. Students should note that they now have two coordinate systems at their disposal. They should realize that for some problems either the rectangular or polar system may be necessary, however, one system is usually preferable over the other. Students should develop the conversion formulas and practice using them. C. Discuss graphs of polar coordinates equations. After a presentation at the board the students should be given a written assignment on polar graphing. D. Present the graphs of a rose curve and lemniscate at the board. On a written assignment ask students to graph a limacon and a spiral. Select two students to present their graphs at the board.	<u>Career</u> Harlandale Audio-Visual Center <u>Commercial Artist;</u> cassette tape -- cas.T-42 <u>For Additional Information on Illustrating Careers</u> National Art Education Association, National Education Association, 1201 16th St. NW., Washington, D.C. 20036.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		provides a good foundation for obtaining employment in commercial art and may be essential for promotion. Salaries vary widely according to ability and experience.

Teaching Activity

Manufacturers of lamps often supply polar curves showing the candle-power distribution of their products. A polar coordinate graph is used in presenting the curve. These graphs are useful for calculating the size of the lamps required in an installation. The following graph has been prepared by an illustrator employed by a lamp manufacturer. Ask the students to state the candlepower at 20° (OA) and 90° (OB) after examining the graph.



POLAR CURVES FOR 100 WATT LAMP

OA -- 52 candelas
OB -- 46 candelas

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
VIII. Parametric Equations	VIII. THE STUDENT SHOULD BE ABLE TO:	<u>Concept</u> Relationship of an ellipse to the elliptical orbit of a space probe as studied by the space technologist.
A. Definition	A. Define parametric equations as equations which state a relationship between x and y in which each variable is expressed separately in terms of a third variable called a parameter.	<u>Performance Objective</u> Given the eccentricity of the orbit of a rocket in space, calculate correctly on a written problem the perigee and apogee of an elliptical orbit of a space probe as studied by a space technologist.
B. Equations and Graphs 1. Circle and Ellipse	B. 1. Given the equations of five circles and five ellipses, graph with 80% accuracy each equation on a polar coordinate system	
2. Path of a Projectile	2. Given the speed at which an object is projected and the angle at which is projected above the horizontal, calculate how far away the object will strike the ground and its greatest height.	<u>General Information</u> Students wishing extra activities should be encouraged to research other careers in space technology. At present the many job opportunities in space science tend to be very promising to the "projected year" of NASA (1990). There premises to be some new job openings as well as vacancies created by loss of personnel in space related careers.
3. Cycloid	3. Given the equation of the cycloid in parametric form, graph the equation on a polar coordinate system.	Examples: 1. Astronauts-(small number needed--salaries classified) 2. Space Technicians Many other careers are closely related to the space industry such as: engineers (electronic, electrical, aerospace, chemical,

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VIII. A. Define a parametric equation by an example. Use two equations in which each variable is expressed separately in terms of a third variable. The third variable is called a parameter, and the equations are called parametric equations. B. 1. Discuss the equations and graphs of a circle and ellipse by using parameters. After presenting examples select several students to work problems at the board. 2. Discuss the path of a projectile in regard to a bullet or rocket. Work several examples at the board using parameters. 3. If possible, use a physical model to demonstrate a cycloid. Use parameters in determining the graph of the cycloid.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		nuclear, mechanical, and industrial are among the larger classifications), mathematicians, geologists, physicists, biologists, and meteorologists. High school students wishing to enter the space industry should prepare with all available math and science courses to prepare for the jobs of the future. In almost all cases a bachelor's degree is needed with a major in the applicable subject area. A master's degree and Ph.D. degree are necessary in many fields. **Teaching Activity** A space probe which has been rocketed into space has an elliptical orbit which has foci identified by the earth and the sun. The orbit has an eccentricity of 0.7. A space technologist has the job of calculating the closest approach (perigee) and farthest approach (apogee) of the space probe. Major axis= V_1V_2 e= .007 perigee= V_1F_1 apogee= V_2F_1

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		$F_1C=1/2\ F_1F_2=1/2(93 \times 10^6$ miles $F_1C=46.5 \times 10^6$ miles $V_1C= \frac{F_1C}{e} = \frac{46.5 \times 10^6}{7 \times 10^{3}} =$ 6.64×10^9 Therefore, V_1F_1 (perigee)= $V_1C - F_1C$=6,640,000,000 - 46,500,000=6,593,500,000 miles And: V_2F_1(apogee)=$V_1C + F_1C$= 6,640,000,000 + 46,500,000= 6,686,500,000 miles.

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CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
IX. Vector Approach to Analytic Geometry	IX. THE STUDENT SHOULD BE ABLE TO:	<u>Concept</u> Relationship of the study of vectors to the specialized training received by an airplane pilot. <u>Performance Objective</u> Given the air speed of an airplane and the speed of a head wind the airplane is encountering, determine the ground speed of the airplane. <u>General Information</u> Students wishing extra activities should be encouraged to research other careers in the airline industry. They may wish to compile and analyze material on work done by the airline industry. Examples: 1) Pilots and copilots (Approximately 27,000 employed in 1970 by scheduled airlines; 1,600 were employed in 1970 by supplemental airlines; and 2,500 were employed in 1970 by the Federal Government. Salaries averaged about $30,000 a year on domestic scheduled airlines and $37,000 a year on international operations.) 2) Flight engineers 3) Stewardesses - Specified qualifications (19 to 27 years old, 5 feet 2 inches to 5 feet 9 inches tall,
A. Review of Rectangular Coordinates	A.	
1. Equations	1. Given ten algebraic equations, graph with 80% accuracy each equation on a rectangular coordinate system.	
2. Vectors	2. Given the coordinates of ten vectors, graph with 80% accuracy each vector on a rectangular coordinate system.	
B. Scalar Components of Vectors	B. Given the coordinates of the endpoints of ten line segments, determine with 80% accuracy the scalar components of each line segment.	
C. Direction Cosines	C. Given two numbers $L=\frac{x_2-x_1}{d}$ and $m=\frac{y_2-y_1}{d}$ where $d=\sqrt{(\Delta x)^2+(\Delta y)^2}$, determine with 80% accuracy if the two numbers are direction cosines by using the formula $L^2+m^2=1$.	
D. Angle between Vectors and Dot Product	D. Given the coordinates of ten sets of vectors, calculate with 80% accuracy the dot product of each set and in turn the angle between each set.	

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IX.	<u>Career</u> Harlandale Audio-Visual Center	
A. 1. Review the rectangular coordinate system by selecting students to demonstrate at the board the different graphs which have been studied throughout the year. 2. Discuss vectors in regard to their applications in physics and navigation. Review graphing vectors before making a written assignment.	<u>Your Future as a Commercial Airlines Stewardess</u>; magnetic tape -- MT-255 <u>Stewardess</u>; cassette tape -- cas.T-28 <u>Commercial Pilot</u>; cassette tape -- cas.T-39	
B. Present and discuss the formula for the scalar product (or dot product) of two vectors as $A \cdot B = \|A\| \|B\| \cos\theta$. Stress that it makes no difference whether θ is positive or negative, since $\cos\theta = \cos(-\theta)$. Point out that vector products are very useful in engineering.	<u>For Additional Infromation on Air Transportation Careers</u> Addresses of individual companies are available from the Air Transport Association of America, 1000 Connecticut Ave. NW., Washington, D.C. 20036.	
C. Explain and illustrate by example that the direction angles of a line are the angles α, β, and γ made with the positive x-, y-, and z- axes. Next explain that the direction cosines are the cosines of the angles. After instruction make a written assignment on determination of direction cosines.	Air Line Pilots Association, International, 1329 E. St., NW., Washington, D.C. 20004.	
D. Make a written assignment on calculating dot products of sets of vectors and the angles between them. Review determinants by alternatively requesting students to use the following on some dot products.		

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		weight not to exceed 140 pounds, and must be in excellen health). There are several thousand openings in this field each year. Salaries
E. Parallel and Perpendicular Vectors	E. Given the direction cosines of three sets of vectors as 0, 1, or -1, determine if the vectors are parallel or perpendicular.	ranged in 1970 from $523 to $800 per month. 4) Mechanics 5) Airline dispatchers 6) Air traffic controllers 7) Ground radio operators 8) Traffic agents and clerks High school students wishing a career with the airlines industry must prove proficiency in their chosen field. A pilot
F. Dividing Line Segments	F. Given the coordinates of the endpoints of ten line segments, determine with 80% accuracy the coordinates of the midpoint of each line segment.	must have a solid background in math with a degree (often in math) from college. Traffic controllers and flight engineer also require a broad background in mathematics. Stewardesses, traffic agents, and clerks must master the proper use of language (foreign helpful). Teaching Activity
G. Complementary Vectors	G. Given five vectors, determine with 80% accuracy each co-vector.	Pilots must often consider the opposing forces their plan encounters in flight. These forces are considered to be vectors by the pilot in making his calculations. Vecto problems of this nature may
H. Area of Triangles and Bar Products	H. Given the coordinates of the vertices of ten triangles, calculate with 80% accuracy the area of each triangle by use of the bar product.	become very complex. A simplified problem follows. A pilot must average a 350 mph ground speed in order to

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$A \times B = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ E. Review the meaning of parallel and perpendicular in regard to vectors. Draw a set of perpendicular vectors on the board. Ask a student to give the measure of their direction angle. Ask another student for the cosine of 90^0. Since the cosine of 90^0 is 0 the students conclude that vectors are perpendicular when their direction cosine is 0. Continue this type of activity for parallel vectors. F. Present and develop formulas for determining the coordinates of the midpoint of a line segment. Ask several students to determine the coordinates of line segments at the board. $x = \frac{x_1 + x_2}{2}$ $y = \frac{y_1 + y_2}{2}$ G. Define a co-vector (complimentary vector) as a vector that is 90^0 counter clockwise of a given vector. Develop and explain that one may find a co-vector by interchanging the scalar components of the given vector and changing the sign of the first component. H. Develop and discuss the bar product in regard to the area of paralleograms and triangles. Present the bar product of two vectors as the determinant of their scalar		

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		reach his destination on time. In making his flight he encounters a 50 mph head wind. The head wind is considered a vector of opposing force against the speed of the plane. The pilot realizes that the opposing vector reduces his ground speed to 300 mph. Therefore he increase his air speed to 400 mph, thus attaining the necessary ground speed of 350 mph.
I. Equation of a Line by Use of Dot Product	I. Given the direction numbers for ten lines, formulate with 80% accuracy the equation of each line.	
J. Vectors in Space	J. Given the vertices of ten triangles in space, express with 80% accuracy the sides as vectors and find with 80% accuracy the length of each side.	
K. Equation of a Plane in Space	K. Given the three points necessary to determine a plane in each of ten problems, formulate with 80% accuracy the equation of each plane.	

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components. Explain that the bar product is the area of a parallelogram and that 1/2 the bar product is the area of a triangle. I. Develop the equation of a straight line by using the dot-product of two vectors. Apply the equation found to that found - by the general rectangular coordinate approach. J. Discuss vectors in space and their scalar components. Discuss the relationship between a vector and a plane to which it is perpendicular. Develop the cross product of two vectors. After instruction select several students to demonstrate determining the length of vectors at the board by using $\|A\|=\sqrt{a^2 + b^2 + c^2}$. K. Stress that any plane may be represented by a linear equation. Develop and discuss the method for determining the equation of a plane when given a point which the plane contains and a vector which is perpendicular to the plane. After instruction select several students to work problems.		

AUDIO-VISUAL SOURCE INFORMATION

TITLE	TYPE	SOURCE	TIME	B/W OR COLOR
Commercial Artist	cassette tape	Educational Progress Corporation		
Commercial Pilot	cassette tape	Educational Progress Corporation		
Geometry in Art	FS	Curriculum Films Inc.		C
Mathematics in Architecture Series	color slides	J. Weston Walch		C
Stewardess	cassette tape	Educational Progress Corporation		
Your Future as a Commercial Airlines Stewardess	magnetic tape	Guidance Associates		
Your Future as a Draftsman	magnetic tape	Guidance Associates		
Your Future as a Surveyor	magnetic tape	Guidance Associates		
Weather Scientists	16mm	United World Films	14 min.	C